Contents

List of contributors ........................................................................................................ iii

Introduction: comparing currency and circulation systems in past societies .................................................................................................................. 1
Dirk Brandherm, Elon Heymans and Daniela Hofmann

Indeterminacy and approximation in Mediterranean weight systems in the third and second millennia BC ............................................................... 9
Nicola Ialongo, Agnese Vacca and Alessandro Vanzetti

Fragmentation patterns revisited ritual and recycling in Bronze Age depositional practice ......................................................................................... 45
Dirk Brandherm

Weight units and the transformation of value: approaching premonetary currency systems in the Nordic Bronze Age ........................................ 67
Lene Melheim

Heads or tails: metal hoards from the Iron Age southern Levant .......... 85
Elon D. Heymans

Weighing premonetary currency in the Iberian Iron Age .................... 105
Thibaud Poigt

Of warriors, chiefs and gold. Coinage and exchange in the late pre-Roman Iron Age ......................................................................................... 133
David Wigg-Wolf

New wealth from the Old World: glass, jet and mirrors in the late fifteenth to early sixteenth century indigenous Caribbean ..................... 153
Joanna Ostapkowicz

Gifts of the gods objects of foreign origin in traditional exchange systems in Palau ....................................................................................... 195
Constanze Dupont
Indeterminacy and approximation in Mediterranean weight systems in the third and second millennia BC

Nicola Ialongo
Agnese Vacca
Alessandro Vanzetti

Abstract

Research on weight systems used during the Bronze Age, prior to the introduction of writing, generally assumes that the widespread use of metal as ‘commodity currency’ eventually resulted in the adoption of widely shared systems of measurement. Many studies aimed at the identification of recurrent weight values as multiples and/or submultiples of theoretical standard units. This approach faces two limitations: 1) the absence of written sources, or at least statistically sound samples, makes it difficult to either validate or reject any reconstruction of prehistoric systems; 2) in the literate Ancient World, different polities usually retained distinct systems. Here an alternative analytical framework is outlined, making use of elementary statistics and cross-historical comparisons, and relying positively on ‘indeterminacy’ and ‘approximation’ rather than on ‘exactness’. Recurrent weight measures can correspond to ‘Standard Average Quantities’, rather than representing arrays of exact multiples/submultiples of given units. By departing from a ‘fractional’ theoretical logic, one can observe that constant exchange practice may have produced the normalisation of ‘tradable quantities’ and that this can happen without necessarily implying the unification of local systems.

Keywords: weight systems, Bronze Age, indeterminacy, approximation, Standard Average Quantities

Résumé

Indétermination et approximation dans les systèmes pondéraux du Méditerranée, pendant le 3ème et 2ème millénaires av. J.-C.

La recherche sur les unités de poids employées à l’Âge du Bronze présuppose qu'une utilisation courante du métal, sous forme de matière première, comme monnaie d'échange permet une généralisation d'échelles de mesures communes. Plusieurs études ont été menées afin d'identifier les valeurs de poids récurrentes représentant des multiples et/ou sous-multiples d'unités de mesure théoriques standardisées. Ces études ont deux limites : 1) l’absence de sources écrites ou au moins d’un échantillonnage statistique fiable, rendant difficile de valider ou de rejeter toute tentative de reconstruction de systèmes préhistoriques; 2) dans le monde ancien les différentes entités politiques utilisent des unités de mesures distinctes qui leur sont propres. Cet article présente une analyse alternative mettant en comparaison divers cas historiques connus avec des statistiques élémentaires, en s'appuyant sur les concepts ‘d’indétermination’ et ‘d’approximation’, et non sur la notion ‘d’exactitude’. Les mesures de poids répétées peuvent
Introduction

The extensive adoption of balances and balance weights since the third millennium BC in the Near East and in the Aegean (e.g. Ascalone and Peyronel 2006; Petruso 1978; Rahmstorf 2003), and in the second millennium BC in the rest of Europe (Pare 1999), has led archaeologists to assume that it was during these periods that widely shared conventional weight systems were first developed, serving as standards for the assessment of economic value. The same general consensus can be recognised in the argument that the circulation of metal was the crucial factor in the spread of conventional weight systems (Pare 2013; Peroni 2006; Renfrew 2008). Moreover, it has been of general interest
to understand whether weight systems were shared, or at least accepted, over wide areas, and if their adoption could be explained as the outcome of a diffusion process (Alberti et al. 2006; Pare 1999; Peroni 1998).

A main problem of research is rooted in the imbalance of the available evidence between literate Ancient Near Eastern societies and preliterate European societies. In the Ancient Near East, the availability of marked and inscribed weights and written records allows for a refined understanding of different yet interconnected standards. In Europe, while the identification of limited sets of proper weights has become generally accepted (Cardarelli et al. 2004; Pare 1999), research is bound to material evidence alone and weight systems are often sought for by assuming that the origin of European standards has to be found outside Europe itself. There is, in this case, a risk of circular reasoning, as the dependence on external standards is assumed as both the question and the proof. This approach has much in common with the beginnings of metrological research: the comparison of different systems of units, in the belief that they were somehow connected to each other, was popular in Ancient Near Eastern studies in the nineteenth century (the so-called ‘comparative metrology’; Ascalone and Peyronel 2006: 17–40; Chambon 2011: 28–38; Powell 1979). Relations were established through the observation of apparent correspondences between different systems and equivalences intuitively defined following a fractional logic. It must be considered, however, that individual metrological systems, in the detailed form in which we currently know them, still took a long time to be identified. In the early 1900s (Viedebantt 1917; 1923; Weissbach 1907; 1916), sharp critiques of this approach were published, which came to the conclusion that ‘comparative metrology could be of value only after the specialised metrologies had created a more secure basis for comparison’ (Powell 1979: 76). In fact, comparative metrology runs the risk of overestimating the value of correspondences when attempting to infer relations between different ‘systems’ whose internal structure has not yet been independently defined (Alberti et al. 2006; Rahmstorf 2010).

Conversion of different systems is not only an analytical problem for the archaeologist, but was also a concern for ancient economic operators. A wealth of ancient texts from the Near East addresses the issue of conversion: the same quantity can be (and in fact is) counted and recorded according to different systems, and rounded down in order to fit ‘exact’ multiples (or submultiples) of any given scale. This raises the question of the indeterminacy of nominal...
weight standards that can be calculated differently according to different systems of measurement. The mina represents, since the Ancient Near Eastern Bronze Age, the most evident example of this practice, providing ‘official’ correlation between different regional systems on the basis of a single shared value. However, as we are going to demonstrate, different local systems happen to match each other at many different values, thus representing a very real difficulty in determining, on the basis of the empirical evidence alone, which system any given ‘standard’ weight properly belonged to.

Ongoing metrological research has produced a substantial advancement in our understanding of weight systems in the Near East and the Aegean between the third and the second millennia BC (Alberti et al. 2006; Chambon 2011; Parise 1970/71; Petruso 1978; 1992; Powell 1979; Rahmstorf 2003; 2006; 2010; Zaccagnini 1984). The fortunate co-occurrence of texts and inscribed weights has provided the ideal framework for the identification of rather ‘exact’ arrays of multiples of given theoretical ‘units’. On the analytical side, the reconstruction of weight systems has been backed by the application of mathematical/statistical methods, thus determining the definition of common procedures, characterised by the adoption of the ‘cosine quantogram analysis’, or ‘Kendall formula’ (Kendall 1974), a mathematical tool for identifying ‘quanta’, or hypothetical units.

Following the success of eastern Mediterranean and Aegean metrologies, research on Bronze Age Europe has begun in relatively recent times to adopt similar analytical tools: such approaches were in effect successful in identifying small sets of likely balance weights (Pare 1999; Rahmstorf 2010). However, the samples are generally too small to provide statistically reliable results (Pakkanen 2011). Since actual balance weights are generally scarce in Bronze Age contexts, European metrologists have often focused on the search for regularities in the weights of disparate classes of metal finds, such as specific object categories (Malmer 1992; Peroni 1966; Primas 1997; Sperber 1993; Sommerfeld 1994), fragmented items and scrap (Peroni 1998; Primas 1997; Sommerfeld 1994), funerary goods (Wiegel 1994), gold objects (Eiwanger 1989; Sperber 1993) and the overall weight of hoards (Tirabassi 1997). Such studies succeeded in highlighting the existence of recurrent weight quantities. While authors recognised that in these cases the object under study was ‘weighed metal’ and not actual balance weights, an appropriate theoretical framework was never developed to address such differences and to properly interpret significant regularities.

We propose that concentrations of weighed metal quantities can be conceptualised as ‘Standard Average Quantities’ (SAQ), which generally conform to the practice of ‘portioning’ goods (Ialongo and Vanzetti 2016); portions are
characterised by an inherent approximation and are not necessarily exact multiples of official weight units. Simply put, metal objects and hoards are not balance weights: while the latter are ‘exact’ tools employed to assess and to assign a ‘value’ against an objective ‘norm’ and have a ‘quantal’ (theoretical) sequence, the former are items made of definite quantities of metal, whose weights are not straightforwardly related to the same quantal sequence. While it can be legitimately assumed that the frequent concentrations of weight values of metal objects and of whole hoards should be related to some sort of normative system, it is not to be taken for granted that these concentrations are directly linked to simple multiples of one unit. Even in ‘international’ exchange, conversion on specific weights does not imply a single shared standard scale, nor strict exactness. Let us imagine two traders, each with his own set of balance weights (e.g. one in pounds and the other in grams); conversions can conveniently take place by agreeing on quantities that approximately correspond to ‘round’ multiples of both units. In the case of pounds and grams, they could agree upon using 1 lb ≅ 450g as a possible link or as a basic incremental unit (1 lb = 453592g and 450g = 0,992lb), thus limiting the theoretical error in both systems of account to less than 1%. It is clear that, based on the material evidence alone, it would be very difficult to determine whether this transaction was made in pounds or in grams, as well as if a single system was shared by traders or not. The intersection between normative conditioning and simple convenience can, in fact, produce ambiguous results; at the same time, however, both these aspects are crucial in the understanding of economic behaviour. The concept of SAQ has been specifically developed in order to address this ambiguity, which we think is linked to the concept of portioning.

This paper comprises four parts. The Ancient Near East is addressed first, showing that, even in high-control contexts, approximation and indeterminacy should be considered together with the definition of exact theoretical weight units; the discussion is supported by the analysis of three different sets of balance weights found at Troy, Byblos and Ebla, dating to the third millennium BC. The European context is approached next. We focus on Pare’s (1999) analysis of European balance weights, which provides the opportunity to address interpretive problems of the convergence of different weight systems. We then shift the focus away from the supposed ‘exactness’ of balance weights and address the problem of ‘weighed metal’. We introduce the notion of Standard Average Quantity (SAQ) as a middle-range tool for the comprehension of shared, ‘culturally significant’ attitudes in assembling quantities of traded goods (portions). Therefore, SAQs allow to address the relation between routine behaviour and theoretical standard scales in transactions where goods are traded in portions. As a contemporary case study to explore the concept of SAQs, we analyze the weights of portioned goods in modern supermarkets.
Finally, we test our assumptions empirically on a sample of weighed metal objects from several Bronze Age hoards from Italy. It is then proposed that it is possible to use approximate values, without relying on exact weight systems, in order to draw significant conclusions about economic interactions in Late Bronze Age Europe.

Materials and methods

The study is based on the statistical analysis of three different sets of material evidence, drawn from diverse historical contexts.

A. The sample of Ancient Near Eastern balance weights was picked from the third-millennium layers at Troy (western Anatolia; 52 items), Byblos (Lebanon; 95 items) and Ebla (inland Syria; 73 items); weight values were derived from Ascalone and Peyronel (2006).

B. The sample of selected Italian hoards comprises 2195 items in total, taken from 62 hoards divided into coherent chrono-geographical groups and dating between c. 1200–800 BC (Final Bronze Age–Early Iron Age). Eight distinct sample groups are singled out (Figure 1). The sample groups are composed as follows:

- Madriolo, single hoard sample group; NE area; FBA; 92 items (Borgna 1992)
- Poggio Berni, single hoard sample group; centre-E area; FBA; 93 items (Morico 1984)
- S. Francesco, single hoard sample group; centre-N area; EIA; 247 items (Montelius 1893: 335; Sorda 1975)
- Tuscany, four hoards sample group; centre-W area; FBA; 279 items (Cateni 1977; Peroni 1961)
- Contigliano, single hoard sample group; centre area; FBA–EIA; 107 items (Ponzi Bonomi 1970)
- Sardinia, 43 hoards sample group; W area; FBA–EIA; 457 items (Ialongo 2011)
- Ardea, single hoard sample group; centre-S area; EIA; 293 items (Peroni 1967)
- SE Sicily, ten hoards sample group; S area; FBA–EIA; 627 items (Albanese Procelli 1993)

Chronological periods are defined according to the Italian chronology (Pacciarelli 2000): Final Bronze Age=FBA; Early Iron Age=EIA. Cardinal directions are abbreviated as capital letters.

---

2 Chronological periods are defined according to the Italian chronology (Pacciarelli 2000): Final Bronze Age=FBA; Early Iron Age=EIA. Cardinal directions are abbreviated as capital letters.
C. The supermarket ‘portions’ sample was gathered in four different shops (three in Rome; one in Trebisacce, Calabria) and comprises 421 weight measurements of packaged items in total (Ialongo and Vanzetti 2016).

The analytical method employed here is designed to address multimodality in frequency distributions of weight values. In fact, archaeological data are often quantitatively limited, thus making it difficult to conduct accurate statistical tests. While some contexts, if considered individually, may suffer from limited numbers, our total sample has an adequate size.

The datasets have been interpolated via the smoothing-spline method to a standard total size of 2048 points for each sample group. Interpolation is a mathematical method to identify new points on the Cartesian plane, assuming that the distribution conforms to a given function. This provides two advantages: 1) it is possible to obtain large datasets that are more easily processed through statistical software (see below); 2) all sample groups achieve exactly the same size, which makes them easily comparable. Following interpolation, data are arranged in a binned distribution; each sample group is divided into two separate but overlapping analytical series, in order for the analysis to be run on consistent orders of magnitude, and the bin width is set accordingly, that is: series 1: 7.5g–403g, bin width=1.9775; series 2: 54g–3000g, bin width=14.73.

Figure 1. Italian hoards considered in this study. Small circles: single contexts; large circles: groups of contexts. 1 Madriolo; 2 San Francesco; 3 Poggio Berni; 4 Tuscany; 5 Contigliano; 6 Ardea; 7 Sardinia; 8 south-east Sicily.

---

3 The interpolated distribution only serves as an aid for the statistical algorithm and is therefore not displayed in graphs.
The binned distributions highlight a sequence of clusters, i.e. ‘peaks’. In order for a peak to be ‘validated’, it is required that the measurements inside it are normally distributed. The normal (or Gaussian) distribution is here assumed to signal the repeated attempts to obtain a predetermined amount, its mean corresponding approximately to the intended value. The multimodal distributions are processed via a statistical software package (Igor Pro 6.05; WaveMetrics Inc., Lake Oswego, OR, USA), through a specific module which detects concentrations that conform to the Gaussian function (‘multi-peak fitting’). The output displays several Gaussian curves over the binned distribution (e.g. Figures 2, 7). The position and width of the curves are consistent with the distribution of non-interpolated data, unlike their height, which is disproportionately enhanced by interpolation; since width and position are the only parameters relevant to our analysis, the height has no scale and can also be displayed as ‘floating’ over bins if required (e.g. Figure 2).

When analyzing the variability of ancient weights, two separate, yet intertwined problems are in question: 1) the ‘margin of tolerance’, indicating the perception of ancient people; 2) the statistical concept of ‘dispersion’, which is an analytical problem resulting from empirical evidence. The coefficient of variation (hereafter CV, i.e. relative standard deviation) is used here as a measure of dispersion in order to address analytically the problem of the ‘margin of tolerance’ of ancient weights, well knowing that other possible causes may contribute to uncertainty (e.g. corrosion, retrieval and preservation biases etc.). The cumulative CV of all the values in a distribution is assumed as an overall measure of normality. Normality is visually assessed, based on the shape of the graph (e.g. Figures 3, 6, 8).

In our interpretive framework, the normally-distributed concentrations of weight values highlighted by the analysis are ultimately taken as an approximation of SAQs, as formerly defined. The method, therefore, was designed to address analytically the repetitive behaviour occurring in economic transactions, where objects are portioned and/or weighed in order to assess their value.

Indeterminacy and approximation in Near Eastern metrology

The seminal works of Thureau-Dangin (1907), Belaieff (1929) and Hemmy (1935) focused on the identification of the Mesopotamian weighing system through mathematical/statistical methods. Without going into the methodological limitations of these early works (cf. Ascalone and Peyronel 2006: 44–6), it can be stated that one of their main results was the acknowledgment that a certain dispersion is always implicit in the empirical distribution of supposedly ‘exact’
weights; dispersion would mainly depend on the lack of technological precision and on the consequent errors in reproducing standardized balance weights. Hemmy (1935) recognised that the Mesopotamian shekel ranged between 8.08 and 8.53g, whereas Belaiew (1929) identified three clusters of values for the mina of the Ur III period, respectively 484.8–498g, 502.2g and 511.8g. Powell (1979) would later refine this argument, stating, based on a total sample of 950 weights, that ‘Mesopotamian precision weights tolerated an inaccuracy of about 3% of the mass of the object being weighed, which accords closely with the range of accuracy indicated for ancient balances’ (Powell 1979: 83). A similar error margin appears, in fact, to have been taken into account even by ancient operators, as can be concluded from ancient texts (Joannès 1989: 127). However, as we are going to illustrate further below, the actual dispersion of balance weights can attain even higher values.

The concept of the ‘propagation of uncertainty’ is generally advocated by scholars dealing with Ancient Near Eastern weight systems. The common approach implies considering units as merely conventional (i.e. theoretical) and recognising that a certain fluctuation is always present (Alberti et al. 2006; Ascalone and Peyronel 2006; Parise 1970/71; Rahmstorf 2010). Sometimes the fluctuation may result in two overlapping distributions, pertaining to two distinct conventional values; such a case represents a common source of indeterminacy. Uncertainty is raised in particular by the impossibility to know a priori whether a given unmarked weight is either a multiple or a fraction of whatever known unit, and often results in the doubtful attribution of certain balance weights to two or more different systems. This can happen particularly when the mass value of a balance weight falls within the ‘margin of tolerance’ of multiples belonging to more than one system. In this respect the analysis of the Aegean and Anatolian Early Bronze Age weights undertaken by Rahmstorf (2010) serves as a typical example. The author analyses a total amount of c. 230 weights from c. 50 sites. Rahmstorf uses the ‘Kendall formula’ in order to detect ‘quanta’ in the distribution of balance weights between 5g and 15g, given a fixed dispersion of ±5% (Rahmstorf 2010 uses ‘deviation’: 89); he also makes use of marked weights to support the identification of possible standards. The detected quanta cluster around certain masses, among which the quantum of c. 9.4g (corresponding to the Levantine shekel, see below) is strongly represented (more than 2/3 of the total sample is assigned to this unit; Rahmstorf 2010: fig. 8.4). Besides the unit of 9.4g, other standards are attested as well, leading the author to ask himself ‘whether there could already have been various units used in the EBA Aegean that, unfortunately, were lying very close to each other, making definite assignment difficult’ (Rahmstorf 2010: 89).
Units

The contemporary use of a multiplicity of units in the Aegean is implied by Rahmstorf’s analysis. However, which is less than certain is which theoretical units precisely lay behind each ‘quantum’. His tentative attribution of such units to different standards, such as the Mesopotamian shekel of 8.3g and the Levantine shekel of 9.4g, has been possible thanks to previous studies of Ancient Near Eastern metrology, and in particular the work by Parise (1970/71; 1981; 1984), who first identified the existence of three different shekels, respectively in use in western Anatolia, the Levant and inland Syria.

Parise reconstructed the conversion rates between the different shekels (i.e. the shekel ‘of Khatti’, the shekel ‘of Ugarit’ and the shekel ‘of Karkemish’) and recognised that the three distinct series possessed a common ‘standard’ multiple in the mina with a theoretical value of 470g (the so-called ‘western mina’), widespread in the Levant alongside the Mesopotamian daric of 500g. He was able to calculate, through a comparative analysis of cuneiform texts and inscribed balance weights from Ugarit (late second millennium BC), the ratio between shekels of different systems and the mina of 470g. The standard value of the mina was defined according to a ratio of 60, 50 or 40 units, characterising, respectively, the Syrian, Levantine and Anatolian systems. During the Late Bronze Age, 60 shekels ‘of Karkemish’, 50 shekels ‘of Ugarit’ or 40 shekels ‘of Khatti’ (with theoretical values of 7.83g, 9.40g and 11.75g) were respectively required in order to obtain a mina of 470g (Parise 1984: 129). Indeed, the widespread adoption of the ‘western mina’ in the Syrian and Levantine areas can be dated back to the mid-third millennium BC (Early Bronze Age), as can be inferred from texts and balance weights discovered at Tell Mardikh/Ebla (Syria), where the three systems are already documented, with substantially the same values as those attested in the late second millennium BC (Arch 1987; Ascalone and Peyronel 2006: 23–5; Milano 2003; Pomponio 1980; Zaccagnini 1984; 1999/2001; see below).

Conversion rates

Conversion rates were often applied in order to facilitate economic transactions in interregional trade. The existence of shared units (i.e. mina and talent) suggests that the mina of 470g functioned effectively as a link between different weight systems. While in each region of the eastern Mediterranean the mina fractions ‘were calculated differently, the bulk quantities of commodity (especially wool and metals) could have circulated without difficulties from one side to another’ (Alberti et al. 2006: 1). The potential confusion deriving from the coexistence of a multiplicity of unit standards sometimes gave rise to the need for specifying
the reference system employed in transactions, such as in the case illustrated by a cuneiform text from Alalakh (AT 33; second millennium BC) (Chambon 2011: 84; Zebb 1991): it is reported that a noblewoman, named Sumunnabi, purchased 135 jars of beer according to the standard ‘of Alep’, for the price of 135 (silver) shekels ‘of Alep’. This example hints at the requirement for the reference system to be specified, in order to state that Sumunnabi paid the price according to the standards applicable at that time in the kingdom of Yamḥad (Alep), and not in some other country (Chambon 2011: 85).

**Approximation and weight-loss control**

The concepts of approximation and margin of tolerance are always to be considered when dealing with ancient weighing systems. These come into play in the weighing practice and are strictly related to both balance technology and weighing procedures (Peyronel 2011). Mari’s texts (second millennium BC) report three types of weighing procedures (Joannès 1989) employing equal-arm balances: 1) the object to be weighed was placed on one pan of the balance, while on the other pan weights were added until the equilibrium was reached (‘simple weighing’); 2) several weights, exceeding the mass of the object to be weighed, were put on one pan, while on the opposite one other weights were added to the object until the equilibrium was reached (‘counterweighing’); 3) when the ‘exact’ equilibrium was not achieved through the previous methods, the weight was approximated (akk. sîqum) and rounded down, within a reasonable margin of error (‘approximate weighing’). The term sîqum (approximation) is often attested. Written records indicate that the margin of tolerance was about one shekel when the object weighed more than one mina and in the range of the shekel’s fraction when the objects weighed less than one mina (Joannès 1989: 139). This implies a perception of the concept of ‘order of magnitude’, albeit possibly still empirical.

The question of approximation often appears as a primary concern in bureaucratic practice and is deeply intertwined with instances of control by central authorities. Mari’s texts report about a specialised officer, named the ebbum, who was in charge of controlling the transactions of metals (Durand 1987). The officer supervised the weighing procedures, often in the presence of the King of Mari. Through the weighing procedures the palace controlled the flow of metals, particularly of those allocated to artisans in order to manufacture prestige objects (Arkhipov 2012: 183). Joannès (1989: 127) observes that officers in charge of supervising the weighing and the value conversion of metals were appointed with the duty to supervise the many steps of the whole process (through repeated weight checks) from the ‘purchase’ of the raw material, through smelting and until the ultimate
shaping, in order to certify that the original mass had not undergone undue weight loss. The officer’s responsibility was further heightened by his awareness that multiple sets of balance weights (belonging to different systems) were contemporarily in use in the palace: ‘La concurrence de plusieurs services de poids pouvait ainsi être source de distorsions, d’où la nécessité de noter leur origine: service du roi, poids du marché’ (Joannès 1989: 127).

**Official units and the multiplicity of measures**

Public administrative institutions, such as palaces or temples, certainly played a major role in the rationalisation of measures by fixing the official standard (Ascalone and Peyronel 2000; 2001). The authority guaranteed the accuracy of measurements through ‘official’ balance weights, stored in public buildings (such as temples and palaces). In the Royal Palace of Ebla (mid-third millennium BC), in addition to weights pertaining to the Syrian system, several other series are documented, likely including the Levantine, Anatolian, Mesopotamian and Aegean systems; this testifies to a multiplicity of measures, simultaneously employed by the palatial institution, in order to account for economic transactions. In this respect, all the balance weights documented in Palace G at Ebla must be considered as ‘official’ weights of the local administrative bureau, regardless of the respective reference systems (Ascalone and Peyronel 2006).

One of the first ‘official’ attempts to reorganise the systems of measures (through metrological linkages of weight, volume and capacity systems) is ascribed to the Akkadian dynasty (2350–2112 BC; Powell 1987/90: 508); however, the first actual metrological reform, leading to the definition of a ‘royal’ standard proper, is introduced slightly later by king Ur-Namma of the third dynasty of Ur (2112–2095 BC). This reform is not likely to have occurred as an introduction *ex novo* of metrological standards, but rather as an official acknowledgment of already existing ones. In his ‘Codex’ Ur-Namma provides a ‘list of equivalences’ and states that he ‘fixed’ the value of a *shekel* at 1/60 of a *mina* (Wilcke 2002). In a more recent analysis of the text a different interpretation of the term ‘fixed’ (sum. *hé-ni-ge-en*) has been proposed: Chambon (2011: 38–40) suggests that the term should be understood as ‘confirmed’ (after Frayne 1997). Therefore, the reform should not be seen as an attempt to impose new standards, but rather to formalise pre-existing ones (Chambon 2011: 41). The reform led to the emission of inscribed weights with royal names, ‘warranting’ the official metrological standards issued by the central authority (Ascalone and Peyronel 2000; Chambon 2011: 40).

---

4 ‘The concurrence of several sets of weights could hence be a source for distortion, giving rise to the necessity to note their origin: service of the king, weight used in the market’ (editors’ translation).
Empirical variability

One of the fundamental assumptions of metrological studies is that concentrations of values represent either actual units of measurement or their multiples. However, the straightforward assumption that approximate weight clusters always represent units, or ‘round’ multiples, can lead to substantially biased interpretations. In order to define our research framework, we empirically tested this assumption against the real distribution of Near Eastern balance weights, and in particular on the rich record provided by the site of Ebla, studied by Ascalone and Peyronel (2006). They attempt a detailed analysis in order to relate each balance weight to its most likely reference system. Ascalone and Peyronel are well aware of the inherent indeterminacy of supposedly ‘exact’ weights, and in fact often provide different likely references for uncertain specimens. Their caution is further supported by our analyses.

The frequency distribution clearly shows that clusters (i.e. ‘peaks’) are indeed well recognisable across the whole series (Figure 2). However, if we look closely at what is actually ‘inside’ the peak, it clearly emerges that almost all significant

Figure 2. Binned distributions of balance weights from Ebla (third millennium BC). Different bar fills indicate different systems (after Ascalone and Peyronel 2006); overlaid ‘peaks’ indicate significant concentrations. Top: values between 0–15g; bottom: values between 10–80g.
concentrations are composed of balance weights belonging to two or more different systems of units. As several authors have remarked (Ascalone and Peyronel 2006; Pakkanen 2011; Parise 1970/71; Petruso 1992; Rahmstorf 2010), the confident identification of specific units is possible in Ancient Near Eastern contexts (and it is safe to reiterate: with a certain degree of uncertainty) only by virtue of the conversion factors provided by the correspondences between texts and inscribed weights, which means that the same array of ‘precise’ weights would be very difficult to identify in other contexts where ancient standard measures are not corroborated by external evidence (e.g. texts). Can we say, then, that clusters represent units? It is clear that the answer is not univocal. In other words: clusters indeed represent units and their multiples, but at the same time each cluster may account for a multiplicity of different systems of measurement.

The problem of approximate clusters also raises the question of the ‘margin of tolerance’. Here, we will use the coefficient of variation (CV) as a measure of dispersion of weight values. While the threshold of ±3% may be assumed as a standard theoretical one (Powell 1979), in practice it may be too restrictive and perhaps not always adequate to address the actual variability of real samples (for instance, Rahmstorf [2010] employed ±5%). The analysis of the weights found in third millennium levels at Ebla, Byblos and Troy will help to clarify this statement. The cumulative CV of the real distributions of balance weights from the three sites (Figure 3), computed according to the standard values identified by Ascalone and Peyronel (2006), shows that: 1) values are normally distributed, which lends strong support to the attributions made by the authors and 2) the real margin of tolerance can attain values of CV as high as ±8%. While the balance weights at Troy and Ebla are in line with the theoretical ±3% threshold,
the distribution of the Byblos sample indicates a much higher dispersion. The peculiar position of Byblos as a major port of trade in the eastern Mediterranean might have been responsible for a much higher frequency of conversion operations between different systems of weights than at Troy and Ebla (Schon 2015, for example, has observed that the dispersion of different sets of balance weights tends to vary in relation to both the sampling strategies and the type of use context); this could have caused a higher level of indeterminacy (or even hybridisation?) across different systems. While the Byblos case might be discarded as an ‘anomaly’, it should nonetheless warn against an overconfident reliance upon strictly predetermined confidence levels.

With this brief aside on Ancient Near Eastern metrology we have attempted to focus on issues of indeterminacy and approximation. What appears to emerge from the discussion is that our uncertainties in identifying the underlying schemes of ancient measures are somehow mirrored in specific issues recurring in ancient practices. While it may be true that ‘exactness’ was the ultimate purpose of accounting for incomes and expenditures, we must bear in mind that such exactness was almost entirely theoretical and that it did respond, in practice, to the necessity of minimising complaints in public or judicially relevant economic transactions (Schon 2015). We chose to focus on public aspects of weighing practices, but official control was also required in order to regulate different instances of private behaviour. This consideration should suggest further caution towards the over-confident application of ‘exact principles’ to pre-state societies, like in Bronze Age Europe, where it is uncertain whether central authorities existed to guarantee for the ‘officialness’ of measures (as already highlighted by Rahmstorf 2010). Furthermore, we have shown how problematic it can be to identify different systems based on balance weights coexisting in the same context. Moving now to Late Bronze Age Europe, we will see the problems related to the widely shared hypothesis that European standard units came into existence at least since the Middle/Late Bronze Age, and that they converged toward the Aegean ones, or were directly borrowed from the Aegean world in a Mediterranean context of increased trade.

**Indeterminacy and convergence of weight systems in prehistoric Europe**

**Premise**

Studies on prehistoric European metrology generally attempt to reconstruct system(s) of measurement through the discovery of its (their) basic unit (Lenerz-de Wilde 1995; Malmer 1992; Pare 1999; Peroni 1966; 1998; Primas 1997; Sommerfeld 1994; Sperber 1993; Tirabassi 1997; Wiegel 1994). The rationale behind the ‘quest for the unit’ follows the assumption that the more or less
widespread adoption of a given unit should account for an equally uniform system of measurement. Regardless of the different methods employed, studies on European Bronze Age weights and possible weight systems share a common approach, recently summarised by Peroni (2006): 1) the trade of metal is the main form of commodity exchange in Europe during the Bronze Age; 2) the progressive diffusion of weighing equipment implies a conscious approach to the quantification of economic value; 3) the frequency in mid-to-long range exchange produces a convergence towards the definition of ‘standard amounts’ of weight, which can be identified in the archaeological record; 4) the convergence towards standard values is the only archaeologically observable parameter that allows to address interconnections between radically different economic systems.

**Margin of tolerance: a measure of indeterminacy**

Pare (1999) recognises a class of rectangular objects in Late Bronze Age contexts (Br D), with a significant distribution in elite burials in central Europe, which he convincingly identifies as balance weights (Table 1). By applying the ‘Kendall formula’, he finds at least three values (3.6, 6.9 and 20.1g) which, he suggests, can work as ‘units’ for the system of measurement to which such weights were meant to conform. Based on intuitive fractional calculations, Pare further proposes a fundamental unit of 61.3g, represented by the weight from Gondelsheim (60.65g). He makes a strong point about his system being substantially analogous to the Aegean one, based on a unit of c. 61g according to Petruso (1992), and argues for an Aegean derivation. This argument seems to be historically deduced, as it fits within the Europe-Mediterranean connections labelled as the ‘metallurgical koine’ of the Late Bronze Age (Peroni 2004). We will try to describe how approximation and variability come into play in the comparative study of different systems of units by discussing Pare’s attempt to connect the central European system to the Aegean one.

The case presents several critical points: 1) the sample (17 items) is far below the required confidence level suggested for quantal analysis (Pakkanen 2011); 2) five weights out of 17 escape the quantal logic and are deliberately left out of the conclusions; 3) the interpretation shows an overconfident reliance on the initial assumption, i.e. that quanta are in fact units, and tends to bypass other possible causes, for instance that quanta can be influenced by clusters in the distribution deriving from the coincidence/closeness of different systems of measurement; 4) this becomes clearer if we consider a simple fact inherent to basic mathematical reasoning, i.e. that ‘the same products of pairs of numbers may be obtained by multiplying vastly many different pairs of factors’ (M. Lo Schiavo 2009).
Now, let us assume such cautious considerations as a mere list of caveats and admit that the distribution of rectangular weights is in fact strikingly similar to multiples of the Aegean shekel (6.69g, according to Zaccagnini 1999/2001). But is the Aegean unit the only possible reference for the central European system? We attempt to tackle this question by approaching the record as a series of distributions, rather than an array of exact values (Table 1). The mass value of each rectangular weight is compared to the closest multiple of every known elementary unit (shekel) in use in different areas of the Mediterranean during the second millennium BC (Ascalone and Peyronel 2006; Parise 1970/71; 1981; 1984; Petruso 1992; Rahmstorf 2010; Zaccagnini 1999/2001); individual measures on the same row are considered as if they were part of the same distribution, and the CV is calculated accordingly. The average CV of each distribution on the same row is generally fairly low and tends to decrease as magnitude increases. Whereas the one-to-one comparison with the Aegean system might appear to support a direct derivation, the perspective changes substantially if we consider all Mediterranean shekels, ultimately providing a more nuanced framework: different systems appear quite easily convertible into one another, provided statistical dispersion is taken into account and kept at a tolerable level. We do not question the affinity of the central European system with the Aegean one; nonetheless, our attempt to extend the comparative framework shows that affinities with other contemporary systems also exist. Furthermore, a close look at the average CV of each system, compared individually with the distribution of rectangular weights, clearly indicates that the Aegean series is not even the most akin, ‘eastern’ Aegean and Levantine series being somewhat ‘closer’ on average and the Syrian one almost on par (Table 1); in fact, only the Anatolian system seems consistently different. We believe that the nuanced framework emerging from our analysis should suggest caution in applying straightforward diffusion models.

Only historical considerations lend support to the proposed derivation from the Aegean, or from the eastern Mediterranean in general, whereas the empirical distribution of measures and the fractional logic appear insufficient to support the hypothesis of a straightforward and precise derivation from a specific system (e.g. the Aegean one). This raises the question of whether a specific diffusion model is the only option to interpret the apparent convergence of the different weight systems throughout the Mediterranean, and even Europe.

‘Convergence’ of different systems of units: an ill-posed problem

The above analysis has shown that different Mediterranean systems appear to converge on similar values, or fractional values, and we have also illustrated the case of the convergence/conversion of the shekel and the mina in the Ancient
Near East. The question of the convergence of different systems, if tackled from its basic principles, can be explained empirically as a simple consequence of the mathematical properties of different series of discrete units, and might even imply a limited relevance of cultural factors.

We have tried to illustrate a synthetic conceptualisation of how multiples of the different Mediterranean units can apparently converge around approximate common values (Figure 4). The Aegean unit of 6.69g is used as the main reference to which other units will be compared and a series of its multiples from ×1 to ×60 is calculated; for each remaining unit, the ‘round multiple’ closest to the obtained Aegean multiples is then calculated and the CV of the values on the same row is computed. It is strikingly apparent that the CV falls very sharply through the lowest multiples (between ×1 and ×3) and stabilises at very low values from multiple ×6 onward. Of course, if fractions were included in the calculations the CV would have stabilised on very low values from the start of the distribution. The following ‘rule of thumb’ can be derived: given the set of Mediterranean units, any value above 40g can always be indefinitely attributed to any system of units, without using fractions. It is safe to reiterate here that such indeterminacy can be dealt with, to a fair extent, in Aegean and Near Eastern contexts, where theoretical systems of fractions and equivalences

<table>
<thead>
<tr>
<th>Mediterranean units</th>
<th>E-Aegean</th>
<th>Aegean</th>
<th>Anatolian</th>
<th>Levantine</th>
<th>Syrian</th>
</tr>
</thead>
<tbody>
<tr>
<td>factor</td>
<td>value</td>
<td>CV</td>
<td>factor</td>
<td>value</td>
<td>CV</td>
</tr>
<tr>
<td>3.83</td>
<td>6.69</td>
<td>2.0%</td>
<td>6.69</td>
<td>1.1%</td>
<td>7.8</td>
</tr>
<tr>
<td>6.5</td>
<td>10.0</td>
<td>2.5%</td>
<td>11.75</td>
<td>2.3%</td>
<td>12.9%</td>
</tr>
<tr>
<td>6.7</td>
<td>11.5</td>
<td>1.1%</td>
<td>11.75</td>
<td>2.7%</td>
<td>10.7%</td>
</tr>
<tr>
<td>7.45</td>
<td>12.6</td>
<td>7.6%</td>
<td>2.2</td>
<td>0.2%</td>
<td>7.8</td>
</tr>
<tr>
<td>7.86</td>
<td>14.0</td>
<td>11.4%</td>
<td>2.2</td>
<td>0.5%</td>
<td>7.8</td>
</tr>
<tr>
<td>10</td>
<td>15.3</td>
<td>2.3%</td>
<td>2.133</td>
<td>8.3%</td>
<td>14.6%</td>
</tr>
<tr>
<td>15.01</td>
<td>15.6</td>
<td>2.2%</td>
<td>2.133</td>
<td>8.3%</td>
<td>15.6%</td>
</tr>
<tr>
<td>15.55</td>
<td>15.6</td>
<td>0.2%</td>
<td>12.33</td>
<td>10.6%</td>
<td>15.6%</td>
</tr>
<tr>
<td>19.89</td>
<td>19.8</td>
<td>3.2%</td>
<td>2.037</td>
<td>0.6%</td>
<td>23.4%</td>
</tr>
<tr>
<td>20.8</td>
<td>20.8</td>
<td>0.0%</td>
<td>2.3</td>
<td>8.6%</td>
<td>23.4%</td>
</tr>
<tr>
<td>21.4</td>
<td>20.8</td>
<td>2.0%</td>
<td>2.3</td>
<td>6.6%</td>
<td>23.4%</td>
</tr>
<tr>
<td>21.45</td>
<td>20.8</td>
<td>2.2%</td>
<td>2.3</td>
<td>6.4%</td>
<td>23.4%</td>
</tr>
<tr>
<td>39.27</td>
<td>41.6</td>
<td>0.1%</td>
<td>40.14</td>
<td>1.5%</td>
<td>35.25</td>
</tr>
<tr>
<td>41</td>
<td>41.6</td>
<td>2.3%</td>
<td>40.14</td>
<td>1.5%</td>
<td>47</td>
</tr>
<tr>
<td>43</td>
<td>41.6</td>
<td>2.3%</td>
<td>6.4014</td>
<td>4.9%</td>
<td>47</td>
</tr>
<tr>
<td>55.02</td>
<td>57.2</td>
<td>2.7%</td>
<td>53.52</td>
<td>2.0%</td>
<td>58.75</td>
</tr>
<tr>
<td>60.65</td>
<td>62.4</td>
<td>2.0%</td>
<td>60.21</td>
<td>0.5%</td>
<td>58.75</td>
</tr>
</tbody>
</table>

Table 1. The mass values of rectangular weights from central Europe (first column on the left, values in grams; after Pare 1999) are compared to the ‘closest’ multiples of each known Mediterranean shekel. For each Mediterranean system, a separate column indicates the CV of each multiple of a given shekel in relation to the ‘closest’ rectangular weight. In the bottom row, the average CV of each series is calculated.
are known *a priori* through texts and their identification is facilitated in practice by the occurrence of marked and inscribed weights. In particular for the Bronze Age Aegean, several studies remark that the validation of mathematically reconstructed series of exact values can only occur if both conditions (i.e. marked weights and the availability of texts as reference) are met in the same array of sample balance weights. Such cases gain strength through the observation of the occurrence of clusters of weight values around the proposed units, fractions and multiples; however, empirical data do not conform to exactness, and interpretation (based on texts etc.) is crucial (Pakkanen 2011; Petruso 1992: 63). It follows that the same indeterminacy can stand as an inextricable puzzle in prehistoric Europe if we try to infer systems of units only through mathematical and comparative means.

The general affinities emerging from the comparison of different Mediterranean series suggest a more complex framework than simply the transmission of a system of account from one ‘country’ to another. As we have shown, the reasons for such affinities can be largely independent from any cultural/historical situation. Simply put, any paired series of units will ‘get close’ to each other indefinite times, in correspondence with approximate common multiples; this is to say that, even if not *exactly* matching, the two multiples will be ‘close enough’ to be considered within the same margin of tolerance (M. Lo Schiavo 2009; Rahmstorf 2010: 89). The only way to validate the fractional logic would be to find strict and recurrent correspondences between ‘relevant’ multiples, and possibly full series of multiples (cf. Schon 2015), but the European sample does not yet allow for the required levels of statistical significance. A typical case of good

![Figure 4. Top: multiples of the ‘Aegean’ shekel (after Zaccagnini 1999/2001) compared to the ‘closest’ multiples of other Mediterranean shekels; in the right column, the CV is computed for all values in the same row. Bottom: the CV of each row is plotted against each distribution mean.](image-url)
correspondence is that already discussed for the Ancient Near Eastern mina, where, however, the interpretation is highly reliant on textual evidence.

**Constructing an alternative frame of reference: approximation, convergence and Standard Average Quantities**

**Standard Average Quantities: a middle-range tool**

Having clarified our critical remarks and cleared the path of what we consider potential sources of bias, we base our enquiry upon the third of the four key points made by Peroni (2006, quoted above), that is, that ‘standard amounts’ of weight are effectively recognisable in the archaeological record. Metrological research on Bronze Age Europe has demonstrated beyond reasonable doubt that a vast array of metal objects, other than balance weights, tend to cluster around recurrent approximate values. Clusters are recognisable not only within the same classes of artefacts (Lenerz-de Wilde 1995; Malmer 1992; Peroni 1966; Primas 1997; Sommerfeld 1994; Sperber 1993), but also across different object categories (Ialongo et al. 2015; Peroni 1998; Wiegel 1994), as well as in fragmented items (Ialongo et al. 2015; Peroni 1998; Primas 1997) and even whole hoards (Ialongo et al. 2015; Tirabassi 1997). Correspondences of values are so frequent, even across wide geographical areas, that they can be best interpreted as the result of intentional behaviour that aims at achieving a predetermined weight quantity.

Our analysis aims at addressing the apparent convergence of different systems of measurement around recurrent values, and is based on the hypothesis that different systems of measurement show convergence around those quantities which, for practical reasons, are most frequently employed in exchange activities. On empirical grounds, it is then assumed that convergence phenomena produce arrays of ‘portioned goods’ whose weight measures tend to cluster in correspondence of such convenient values.

In practical terms, as we have shown, the two aspects cannot be easily separated and distinguished from each other in the archaeological record. On the other hand, the significance of clusters of values in the weight of objects is likely due to both such qualities, and perhaps keeping them separate will not result in a profitable approach. This is all the more true for prehistoric European contexts, where a straightforward identification of exact theoretical units is still highly uncertain. The concept of ‘Standard Average Quantity’ (SAQ) was designed to account for such a duality and is meant to provide the middle-range tool to connect empirical observations to the broader interpretive framework. A SAQ can be defined as follows: a recurrent, conveniently tradable quantity (of mass), whose adoption is acknowledged within one, or across several different cultural
systems. In this respect, a SAQ has a similar function to that of the *mina*, since both are meant to link together different standard systems and to provide utility in conversion operations (Parise 1970/71; 1981; 1984). A SAQ can be measured in terms of any existent system of units, as its utility depends on the agreement between economic actors. A SAQ is, therefore, a ‘practical unit’ according to which goods are, in effect, portioned and traded. SAQs are ideally represented, on empirical grounds, by the statistically significant concentrations of weight values. In fact, SAQs are not exact, their approximate nature being due to either measure imprecision or inconsistency of different scales of measures. In our model, SAQs are not independent from normative (i.e. theoretical) systems of measurement, but are dynamically involved with them in a dialectical relation, reciprocally shaping each other. In the following paragraph we try to illustrate this process, using a case study different from that of balance weights.

**A contemporary case study for the normative qualities of SAQs**

To proceed by analogy allows us to extend the framework to more recent times. We will first introduce the role of SAQs with a modern case of weight-related economic behaviour, i.e. the definition of the ‘oil barrel’ as a unit. The discussion will then focus on the results of recent research on the recurrence of weight values in portioned goods in modern supermarkets (Ialongo and Vanzetti 2016). We attempt to show that, even in modern economies, there is much room left for approximation and ultimately for SAQs to be brought into common use. As a consequence, SAQs have a relevant role in shaping ‘customary’ economic habits, and this role is at least partly independent from official units sanctioned by central authorities.

In 1866, US oil producers set up an agreement and established the standard quantity of the unit of measure, still employed in the US in present days, commonly known as the oil ‘barrel’. Until then, in the early years of oil extraction in the US, a specialised container was yet to be introduced and oil was shipped in reused wooden barrels, originally containing the most disparate goods (from fish to whiskey) and averaging 42 gallons in volume capacity (around 160 litres, allegedly ‘as much as a man could reasonably wrestle’). According to the American Oil & Gas Historical Society (AOGHS 2013), the boom of oil production in the early 1860s caused the whole available stock of wooden barrels to be almost wiped from the market: it was in such circumstances that specialised containers were first produced for the oil market, their standard capacity being eventually established at 42 gallons.

This is just an anecdote, yet it provides a suggestive glimpse on how units of measurement can actually come into being out of customary behaviour, even
in the industrial era; it renders with a certain precision what SAQs consist of in our model. The first thing one can note is that the ‘standard quantity’ was already in use before it was officially acknowledged as a unit of measurement proper, similarly to how Ancient Near Eastern reforms ratified pre-existing standards. From an organisational standpoint, the ratification of the 42 gallons barrel was driven by convenience, as sellers and buyers alike were already familiar with the average quantity which the product was shipped in; therefore, making an already customary measure the ‘official’ one would have likely appeared as the most convenient choice for all the agents involved. Put in other terms, the ratification of the unit of measurement intervened in formally regulating a specific instance of market exchange which already had its customary norms, and which was, in turn, already regulated by a well-established, relational framework of habit and trust. The idea that official units may derive from customary standards is not new. Lenerz-de Wilde (1995, followed by Pare 1999; Peroni 1998; Primas 1997) made a convincing argument about the earliest European standards having derived from widely distributed ingot-like objects, such as rings, torcs and axes, between the Late Copper Age and the Early Bronze Age. For the Ancient Near East, Powell (1987/90) suggests a shared etymology of the term shekel and the Sumerian word for ‘axe’, hinting that the term could have initially referred to axes as approximate standards. Moreover, the Sumerian, Akkadian and Greek words for talent would all basically mean ‘burden/load’ (Powell 1987/90: 510), hinting that a talent would stand for ‘as much as a man can carry’ (Ascalone and Peyronel 2006: 42); this, in turn, closely recalls the origins of the oil barrel, stemming from recycled containers and reportedly selected in order to contain ‘as much as a man could reasonably wrestle’. The concept of a relationally defined convergence process is crucial to our model of SAQ and embodies an alternative view in respect to the diffusion model that is often assumed in metrological studies. We have proposed that material evidence provides insufficient support for a straight derivation of European units from central-eastern Mediterranean standards (as proposed by Pare 1999 and Cardarelli et al. 2004; see also F. Lo Schiavo 2006; Ruiz-Galvez Priego 2000). The latter interpretation, moreover, owes much to centre-periphery models, which recent research tends to question, suggesting instead a more complex and dynamic framework (e.g. Broodbank 2013; Jones et al. 2014; Jung and Mehofer 2013).

An analytical approach to the formation of SAQs requires highly controlled samples in order to avoid the production of post-hoc arguments. Our sample is consequently picked from a specific form of economic behaviour very familiar to all of us, which we believe stands as a peculiar example of how customers’ desires interact with commercial offer in producing ‘customary standards’: packaged goods in supermarkets. We assume that groceries are assembled in
packages of different quantities in order to meet the preferences/needs of different categories of customers (e.g. singles, couples, large families and so on); therefore, we expect that the distribution of weight values follows an observable multimodal distribution. The full results of the analyses of this supermarket sample have been published elsewhere (Ialongo and Vanzetti 2016); we will focus here on a few specific aspects.

The study was conducted on packaged goods sold in supermarkets in Rome and in Trebisacce (CS, Calabria). The sample is divided into two categories: packages of goods bearing an exact, 'round' nominal weight (with no 'real' weight listed on the label) and portioned goods with their 'real' weight reported on the package. We did not consider any case of goods simply sold by number. To clarify: there is a wide array of packaged goods (e.g. potatoes, onions, carrots) that are sold by a 'round' nominal weight (i.e. 100g, 200g, 500g etc.) and priced accordingly (i.e. with a fixed price), and other goods that are packaged, but priced according to their actual quantity (e.g. meat). Several ‘clusters of values’ result from the analysis (Table 2). In this case, the results are rather easily obtained: it is sufficient to group the different kinds of packaged goods by their respective labels, and then compute the basic statistics of each grouping.

<table>
<thead>
<tr>
<th>Calabria_SAO</th>
<th>Rome_SAQ</th>
<th>nominal value</th>
<th>CV</th>
<th>groceries</th>
</tr>
</thead>
<tbody>
<tr>
<td>140.09</td>
<td>178.32</td>
<td>178.89</td>
<td>150</td>
<td>13.4% garlic, entrecôte, ham</td>
</tr>
<tr>
<td>442.80</td>
<td>408.00</td>
<td>436.20</td>
<td>458.00</td>
<td>470.83</td>
</tr>
<tr>
<td>531.15</td>
<td>540.67</td>
<td>538.00</td>
<td>500</td>
<td>1.6% onion, bacon, pork steak, tomato, sausage</td>
</tr>
<tr>
<td>552.00</td>
<td>610.67</td>
<td>691.11</td>
<td>693.50</td>
<td>600</td>
</tr>
<tr>
<td>671.63</td>
<td>763.33</td>
<td>771.87</td>
<td>750</td>
<td>0.6% onion, apple</td>
</tr>
<tr>
<td>771.87</td>
<td>966.86</td>
<td>1021.05</td>
<td>1054.00</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 2. Actual weight values (in grams) of SAQs in supermarkets in Calabria and Rome (n=421), compared to the nominal values listed on packages (after Ialongo and Vanzetti 2016); the CV of each SAQ is provided. The right column lists the different types of groceries included in each SAQ.
SAQs in supermarkets indeed show clusters around recurrent values, regardless of the nature of the good being portioned; very different groceries are intentionally assembled in order to match a few recurrent, predetermined quantities, which are in turn very often ascribable to approximate ‘round’ multiples of either 50g, 100g or 150g (Table 2). The fact that, in each distribution, at least one package with ‘nominal’ value is always present makes it quite easy to set the indicative equivalence with such round quantities. It is clear that different groceries cluster around the same ‘nominal’ values: with regard to the behavioural/normative duality of SAQs, we interpret such a state of things as the outcome of the intersection between customers’ desires (as they are ‘interpreted’ by the seller) and a form of ‘normative way of thinking’, tending to direct the practice of assembling packages towards ‘round’ amounts, with the simple purpose of facilitating accounting operations for the convenience of both buyers and sellers.

A closer look at the separate distributions of supermarkets in Rome and in Calabria will help clarify how, in our model, SAQs can be strongly influenced by cultural factors that are only loosely constrained, rather than entirely determined by the need to comply with an officially sanctioned normative system. The graph in Figure 5 shows the binned distributions at a very low resolution. ‘Small’ quantities are far more recurrent in Rome than in Calabria, which should mean that there is less demand for small SAQs in Calabria than there is in Rome, at least in the explored supermarkets. Among all possible causes, one that perhaps appears rather compelling is the different composition of households in the two regions: according to the 2011 census (ISTAT 2011), households in Calabria are c. 1.4 times larger, on average, than they are in Rome. Other causes may also come into play, such as the common habit of storing/hoarding food in southern regions of Italy and, in a broader perspective, a substantially lower GDP. Given the limited scope of the analysis, the validation of such a relationship clearly requires both more insight and a larger sample. However, we believe that the significance of the point at stake is sufficiently clear: the formation of SAQs is uneven, and influenced by convenience, profit and other factors, not only by the official norm. On the other hand, evident connections between the two ‘regional series’ (Rome and Calabria) also exist. The two different series match at several significant values and also maintain the same approximate ‘modules’ (one could say ‘quanta’), but produce, in practice, two different empirical distributions, which are in turn available for evaluation through simple statistics. Norm-independent factors such as kinship relations (and perhaps social organisation in general, wealth distribution, ritual habits, disposition to warfare etc.) could shape SAQs at any time, while at the same time being rationally organised in order to approximately match official standards.
Further conclusions can be drawn from the sample and serve to better outline the analogy between supermarket SAQs, balance weights and specifically (as we will discuss later on) metals in prehistoric hoards. First of all, the weights of goods belonging to each type of groceries are normally distributed (Figure 6). Secondly, the cumulative CV is rather high even in packages with nominal values (7% for packages with nominal values; 15% for those without). While this is the consequence of merely practical causes (e.g. grocery SAQs are made of indivisible modules: one cannot sell four and a half tomatoes), it warns us to be cautious in assuming exactness as the only ordering principle, even in our supposedly ‘exact’ economy. Thirdly, it is ultimately impossible to infer the ‘official standard unit’ (i.e. ‘1’ or ‘10’) from the empirical distribution alone, but rather the approximate ‘quantum’ of 50g would appear as significant.

**SAQs in Italian hoards: distribution and correspondences**

The main body of this article was primarily meant to describe our analytical framework and its interpretive implications; the following outline of our preliminary analysis of Italian hoards will serve to illustrate the possible applications of the method and to suggest further developments.

![Figure 5. Binned distribution (interval: 380g) of SAQs in supermarkets in Rome and Calabria.](image)

![Figure 6. Cumulative binned distributions of CVs of the two different categories of SAQs in supermarkets. Top: goods with exact ‘round’ nominal weight listed on the package; bottom: portioned goods with real weight listed on the package.](image)
From the analytical perspective, the main points emerging from our discussion support the following hypotheses to be used in the study of Italian hoards: 1) the distribution of weights is expected to be multimodal; 2) significant clusters of weight values, representing SAQs, should be normally distributed; 3) the CV of significant clusters should fall within a tolerable level (i.e. not more than 8%). We matched our hypotheses against eight different sample groups, each representing either a single, important hoard or corresponding to a group of hoards from a well-defined chrono-geographical context. All items contained in hoards were considered in these analyses, without any selection based either on shape or function.

The results of the ‘multi-peak’ analysis (Figure 7) show that all the distributions are multimodal, and that several ‘peaks’ match across different contexts (Table 3). Since the distributions are continuous, the boundaries of each peak must be set arbitrarily: the graphs in Figure 8 represent the cumulative CV of the sampled distributions, ‘truncated’ respectively at ±5%, ±10% and ±15% from the mean value of each peak and corresponding to CVs respectively equal to 3%, 5% and 8%. Graphs 1 and 2 retain a symmetrical, roughly bell-shaped curve, whereas graph 3 shows the incipient emergence of two more peaks on both sides of the central one (i.e. a multimodal distribution), meaning that the range is large enough to encompass adjacent peaks and should therefore be discarded. Graphs 1 and 2 suggest that the sample is organised according to a multimodal distribution of normally distributed clusters whose CV ranges between 3%–5%, which is in line with the expectations; moreover, the sample truncated at ±5% includes 31% of total measurements, while the ±10% one accounts for 49% of total measurements. To summarise, the results match our hypotheses: we can conclude that a large part of the sample (between 30%–50%) can be explained as an array of normally distributed clusters with a CV between 3%–5%. In the light of the

Figure 8. Cumulative binned distributions of CVs of SAQs in Italian hoards, symmetrically truncated at different distances from the mean (±5%, ±10%, ±15%).
Figure 7. Binned distributions of weight values pertaining to metal objects in selected Italian hoards; overlaid 'peaks' indicate significant distributions. Left: values between 7.5–403g; right: values between 54–3000g (note that the hoard from Crotone lacks items above 400g).
considerations formulated so far in this paper, we could then state that a large part of the metal objects in Italian hoards between the Final Bronze Age and the Early Iron Age were possibly intentionally produced/fragmented in order to obtain a predetermined weight.

The observations on the overall distributions support the idea that the achievement of a predetermined weight was a primary concern, as reflected by objects found in hoards, but provide no information on the weight systems and units in use and on how different systems may possibly have been connected. One could have tried to analyse in more depth the correspondences between different weight series and attempt to devise, by adopting a ‘quantal’ approach, the ‘units’ underlying each system. However, we will show that another approach is possible, making use of indeterminacy and approximation, rather than dismissing them as weaknesses. Following the considerations on the relational qualities of SAQs, we set a preliminary framework aimed at quantifying the potential ties between each chronogeographical context. The mean values are matched in rows (Table 3) in order to obtain a plausible CV for each row. The resulting contingency table was used to run a cluster analysis, in Euclidean distance, choosing the furthest neighbour joining method (the one that most emphasises differences between variables, thus producing more compact clusters; cf. Mooi and Sarstedt 2011: 251–2).

The results are encouraging (Figure 9): the analysis singles out two neat clusters, the first one including hoards from the central Tyrrhenian area (Sardinia, Tuscany and Ardea), and the second one tying together hoards of the central-northern Apennine area (Poggio Berni, S. Francesco and Contigliano) (cf. Figure 1). The clusters appear entirely plausible, grouping together hoards from relatively circumscribed areas; the first cluster, in particular, links together two macro-regions (Sardinia and the central Tyrrhenian area) that are well known to have maintained frequent overseas relationships through the whole of the Late Bronze/Early Iron Age. A third, weaker link is highlighted between south-east Sicily and Madriolo (north-eastern Alps) that may seem to be at odds with the considerable distance separating north-east Italy from the southern Ionian Sea. However, recent research has shown that there is a high probability that a great deal of the metal in use in Adriatic and Ionian Italy (especially in Calabria) during the Late Bronze/Early Iron Age was actually imported from the eastern Alps (Jung et al. 2011). We further observed that, by changing the joining method of the cluster analysis, the basic clusters remain substantially unchanged, thus supporting the solidity of the pattern underlying the data. At this stage of research, this can only suggest that weight systems in the Ionian Sea may have converged towards similar
Table 3. Contingency table of the correspondences between the distribution means of each ‘peak’ from different contexts. The CVs of the distributions in each row are listed in the right column.

<table>
<thead>
<tr>
<th></th>
<th>Madriolo</th>
<th>S. Francesco</th>
<th>Poggio Berni</th>
<th>Tuscany</th>
<th>Contigliano</th>
<th>Sardinia</th>
<th>Ardea</th>
<th>Sicily</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Madriolo</td>
<td>11.45</td>
<td>15.41</td>
<td>19.37</td>
<td>35.19</td>
<td>40.03</td>
<td>49.03</td>
<td>49.03</td>
<td>68.80</td>
<td>...</td>
</tr>
<tr>
<td>S. Francesco</td>
<td>33.21</td>
<td>33.21</td>
<td>39.14</td>
<td>49.03</td>
<td>49.03</td>
<td>49.03</td>
<td>49.03</td>
<td>68.80</td>
<td>...</td>
</tr>
<tr>
<td>Poggio Berni</td>
<td>54.96</td>
<td>54.96</td>
<td>54.96</td>
<td>47.05</td>
<td>49.03</td>
<td>49.03</td>
<td>49.03</td>
<td>68.80</td>
<td>...</td>
</tr>
<tr>
<td>Tuscany</td>
<td>76.70</td>
<td>76.71</td>
<td>82.65</td>
<td>84.62</td>
<td>76.71</td>
<td>82.65</td>
<td>82.65</td>
<td>76.71</td>
<td>...</td>
</tr>
<tr>
<td>Contigliano</td>
<td>108.35</td>
<td>108.35</td>
<td>102.42</td>
<td>108.35</td>
<td>108.35</td>
<td>108.35</td>
<td>108.35</td>
<td>108.35</td>
<td>...</td>
</tr>
<tr>
<td>Sardinia</td>
<td>136.04</td>
<td>143.95</td>
<td>124.17</td>
<td>124.17</td>
<td>124.17</td>
<td>124.17</td>
<td>124.17</td>
<td>124.17</td>
<td>...</td>
</tr>
<tr>
<td>Ardea</td>
<td>187.45</td>
<td>183.50</td>
<td>179.54</td>
<td>179.54</td>
<td>179.54</td>
<td>179.54</td>
<td>179.54</td>
<td>179.54</td>
<td>...</td>
</tr>
<tr>
<td>Sicily</td>
<td>262.60</td>
<td>238.87</td>
<td>230.96</td>
<td>230.96</td>
<td>230.96</td>
<td>230.96</td>
<td>230.96</td>
<td>230.96</td>
<td>...</td>
</tr>
<tr>
<td>MATRIOLO</td>
<td>385.20</td>
<td>381.25</td>
<td>264.58</td>
<td>264.58</td>
<td>264.58</td>
<td>236.89</td>
<td>236.89</td>
<td>236.89</td>
<td>...</td>
</tr>
<tr>
<td>Sardinia</td>
<td>436.98</td>
<td>436.98</td>
<td>306.10</td>
<td>306.10</td>
<td>306.10</td>
<td>306.10</td>
<td>306.10</td>
<td>306.10</td>
<td>...</td>
</tr>
<tr>
<td>ARDEA</td>
<td>540.09</td>
<td>540.09</td>
<td>268.53</td>
<td>268.53</td>
<td>268.53</td>
<td>268.53</td>
<td>268.53</td>
<td>268.53</td>
<td>...</td>
</tr>
<tr>
<td>CONTIGLIANO</td>
<td>775.77</td>
<td>702.12</td>
<td>300.17</td>
<td>300.17</td>
<td>300.17</td>
<td>300.17</td>
<td>300.17</td>
<td>300.17</td>
<td>...</td>
</tr>
<tr>
<td>POGGIO_BERNI</td>
<td>967.26</td>
<td>981.99</td>
<td>339.72</td>
<td>339.72</td>
<td>339.72</td>
<td>339.72</td>
<td>339.72</td>
<td>339.72</td>
<td>...</td>
</tr>
<tr>
<td>S. FRANCESCO</td>
<td>1217.67</td>
<td>1291.32</td>
<td>540.09</td>
<td>540.09</td>
<td>540.09</td>
<td>540.09</td>
<td>540.09</td>
<td>540.09</td>
<td>...</td>
</tr>
</tbody>
</table>

Figure 9. Hierarchical tree-clustering of selected Italian hoards, based on the contingency table in Table 3 (Euclidean distance, furthest neighbour method).
standards to those in north-east Italy, as a consequence of frequent trade along Adriatic routes; in any case, by looking at the whole pattern, results seem to encourage further insight into the possible connections highlighted by the similarities in the distribution of SAQs. To summarise the results of such preliminary analyses, we can conclude that SAQs, as conceptual tools, appear promising in explaining the patterned variation of weight regularities across long distances and in providing interpretive hints that go beyond the mere normative aspects addressed by traditional metrology.

Conclusions

Our discussion of some basic statistical properties of the distributions of weight measures, drawn from Ancient Near Eastern metrology, tested on European samples and discussed against modern proxies, can be summarised in four general statements:

1. a certain dispersion in weight measurements is ‘socially accepted’ in transactions
2. customary standards exist (SAQs) that are only partly related to ‘official’ weight systems and units
3. ‘official’ weight standards (theoretical units) are difficult to recognise through empirical methods alone, while they can be inferred through circumstantial evidence (e.g. texts, marked weights, historical considerations etc.)
4. SAQs are recognisable through empirical methods

If compared to theoretical units (very difficult to determine), SAQs provide an alternative framework in studying the relationships within and across preliterate economic systems, since they can be effectively observed, measured and compared to a much greater extent. But what are we actually observing when we put our focus on SAQs? While theoretical units give us a glimpse on how quantities were counted, and ultimately transcribed in official accounts of literate societies, SAQs are sources of information on more practical aspects of trade activities. The process by which a SAQ comes to represent a customary standard is partly independent from officially-sanctioned units. For example, this is the case of portions in supermarkets, ideally assembled in order to match the needs of average kinds of customers, possibly depending on different compositions of households, but also on cultural practices in storing and shipping goods (as in the case of the modern oil barrel), etc. In general, SAQs, while depending on measuring things and even using theoretical units for formal definition, can be independently used as a source of information on society, economy and trade networks.
Our attempt to analyse the geographical distribution of SAQs in hoards from Final Bronze Age/Early Iron Age Italy through cluster analysis seeks to quantify the connections between local contexts, based on the assumption that similar arrays of SAQs should be related to effective trade relations in a network. The analysis would certainly require a wider scope, both geographically and chronologically; however, the preliminary results are encouraging in that they indicate that the distribution of SAQs might correspond to significant exchange networks in Bronze Age Italy. Our analyses based on SAQs have attempted to show that ‘substantive’, local-level social constraints (in the ‘Polanyian’ sense) are coherent with the mechanisms of large-scale and long-distance exchange. Customary standards, in the form of SAQs, tend to stem from the intersection between ‘social’ and ‘economic’ instances, and their potential to draw observations about large-scale economic networks and trajectories is worth exploring, without necessarily relying on exact weight standards and directional diffusion hypotheses.

Acknowledgements

N. Ialongo devised and coordinated the research, developed the statistical method and performed the analyses; A. Vacca treated Ancient Near Eastern weight systems; A. Vanzetti advised the research process. The theoretical framework was defined by N. Ialongo and A. Vanzetti in previous research and further developed in this study together with A. Vacca. The interpretive framework and the conclusions are a result of the discussion within the research team. We thank M. Ortolani for the support on statistical analyses. We finally thank the editors for providing accurate comments on the manuscript, and giving us relevant suggestions to improve the text; all remaining shortcomings are our own responsibility.

Bibliography


Pakkanen, J. 2011. Aegean Bronze Age weights, chaînes opératoires and the detecting of patterns through statistical analyses. In A. Brysbaert (ed.),


